Global Dynamics in the Solar System

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Abstract. Using Laskar's Frequency Map Analysis, we have performed a complete study of the dynamics of massless particles in the Solar System, from Mercury to the outer parts of the Kuiper belt, for all values of the eccentricities. This provides a complete dynamical map of the Solar System which is, in this first step, restricted to mean motion resonances. The same method is also applied to the planetary 3-body system composed of the Sun, Jupiter and Saturn.

1. Introduction

We present here a unified vision of the dynamics of particles in the Solar System, obtained by the construction of a complete cartography of the Solar System dynamics, using Laskar's Frequency Map Analysis (FMA) (Laskar, 1990, 1999). The main advantages of this approach, besides having a rigorous support (Laskar, 1999), is to permit very short integration times of a few million of years and to identify easily the various involved resonances. As we used only a very short integration time, we could include a huge amount of initial conditions, which provides a complete view of the single particle dynamics in the Solar System. In a first stage, which is presented here, we limit ourselves to the consideration of short term dynamics, or more precisely to diffusion and chaotic behavior resulting essentially from short period resonances. We apply next the same method to planetary 3-body problem.

2. Frequency map analysis

For an Hamiltonian system on $\mathbf{R}^n \times \mathbf{T}^n$, FMA ,constructs numerically a map which associates the n-dimensional frequency vector to the action-like variables (see Laskar, 1999 for details). The dynamical behavior of the Hamiltonian system is obtained from the study of the regularity of this frequency map. The construction of the frequency map requires only a very short integration time and allows to get a measure of the diffusion of the trajectories. This diffusion, corresponding to the variation of the fundamental frequencies with respect to time is computed in the following way: we first determine the frequencies in the time interval $[0, T]$ and then in $[T, 2T]$. For a quasiperiodic solution, the two frequency vectors are equal, and if it is not the case, their difference gives an estimate of the chaotic diffusion of the trajectory over the time T.

3. Short time dynamics of test particles in the Solar System

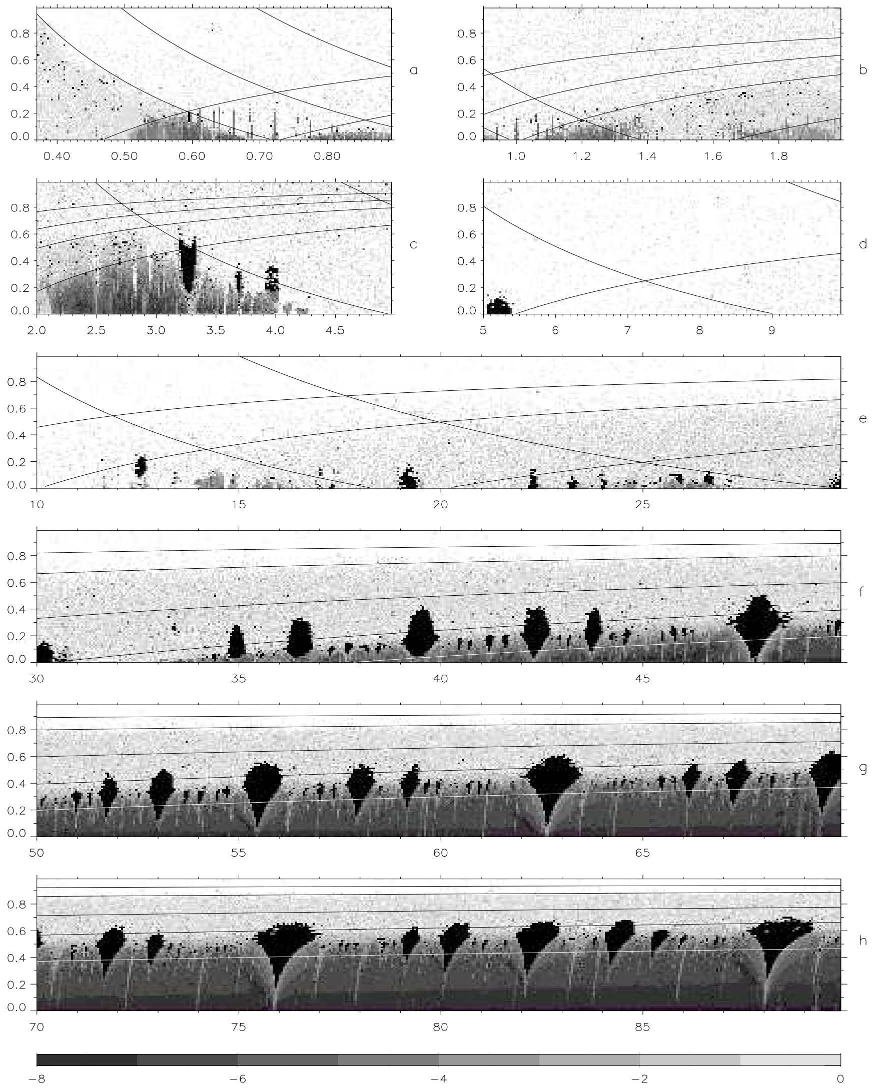

Figure 1. Frequency map of the Solar system in the plane: (a_0, e_0) for $i_0 = M_0 = \omega_0 = \Omega_0 = 0°$.The color code is for $\log \sigma$ (unit: Ma^{-1}).

We consider the motion of test particles in the gravitational field generated by the Sun and N planets (see Robutel and Laskar, 2001 for more details). As the motion of the massless body does not affect the planetary motions, the planet frequencies are fixed, and it is sufficient to study the frequency map in a 3-dimensional space of action-like variables. For any particle, these 3 action variables are the usual elliptical elements (a, e, i) (semi-major axis, eccentricity, and inclination), while the 3 anglevariables are the associated angles (M, ω, Ω) (mean anomaly, argument of perihelion, and longitude of the node). We fix for $t = 0$ all the initial angles of the particle $(M_0, \omega_0, \Omega_0)$ (see Laskar, 1999), and construct numerically the frequency map which associates the numerically determined (n, g, s) frequencies (associated to mean longitude, perihelion and node) to the initial actions (a_0, e_0, i_0). We are only interested here in the short time dynamics, so only the mean motion frequency n will be determined (the two others are associated to the secular motion). This frequency will be called the "proper mean motion", In the sense that if the motion is quasiperiodic, this quantity is a integral of the motion. Moreover, we will fix the initial value of the inclination i_0, and will thus consider only the map $(a_0, e_0) \longrightarrow n$. If the motion is not quasiperiodic, the two values of $n^{(1)}$ and $n^{(2)}$ obtained over the consecutive intervals $[0, T]$ and $[T, 2T]$ will not be equal in general, and the quantity $\sigma = 1 - n^{(2)}/n^{(1)}$ is computed to provide a measure of the diffusion rate of this trajectory.

The results of this investigation are plotted in Fig.1. The initial conditions for the massless particles are taken in the plane defined by $i_0 = M_0 = \omega_0 = \Omega_0 = 0$, and the initial conditions (a_0, e_0) are chosen on a grid. The test particles in the inner Solar System Fig.1.a-c have been integrated with the eight planets (Pluto is not considered) during 0.5 Ma, while the outer system Fig.1.d-h was integrated during 2 Ma taking into account only the four giant planets. For a given particle, three different dynamical situations can occur: First, the particle is ejected or undergoes a close encounter with a planet before the end of the integration; the corresponding dot is plotted in white. Second, the particle survives the integration but is in mean motion resonance with a planet: a black dot is plotted. Third, the trajectory is not resonant: The dot is then colored according to its proper mean motion diffusion rate, from dark grey , for motion close to quasiperiodic ($\sigma \leq 10^{-6}$), to light grey, for strongly irregular motions ($\sigma \geq 0$). The collision lines with the planets correspond to the black curves. This picture provides a striking global view of the dynamics in the Solar System, and emphasizes the separation of the system in three dynamical structures connected and interpenetrated by the zones of mean motion resonances. Contrarily to the inner Solar System (Fig.1.a-c) and the outer planets region (Fig.1.de), these structures appear very clearly in the Kuiper Belt (Fig.1.f-h). Indeed, the outer planet region is too chaotic to contain large regular structures, and the spatial resolution used in the inner part of the system is too coarse to detect resonances with terrestrial planets. We will only briefly describe here these structures in the Kuiper belt.

The first zone is the collisional region, which lies above the collision line with Neptune. Its main characteristic is its strongly irregular dynamics. The particles that have not been ejected over 2 Ma have a very high proper mean motion diffusion rate. This strong chaos is mostly due to close encounters with the planets. The bodies that remain after the 2 Ma integration will probably be removed later from the Solar System after planetary close approach. The only exception to this situation is, as for Pluto, when the particle evolves in the stable area associated to a resonance with Neptune. This situation provides a mechanism which prevent the particle from collision with the considered planet. This is why many resonances penetrate the large chaotic zone above the collision line. The second region named resonance overlap region is perhaps most interesting. This domain corresponds to a mean motion diffusion rate between $10^{-2.5}$ and $10^{-1.5}$ (between the Neptune's collision curve and the white curve in Fig.1.f-h). Fig.1 clearly shows that the resonant zones intersect in this band. Here the diffusion, which is smaller than in the collisional region, is essentially driven by resonance overlap. The chaotic motions induced by this phenomenon will probably lead to a complete depletion of this area by close encounters. As in the collision domain, this region will certainly be cleared after a sufficiently long time. The last dynamical domain pointed out in our study is the region of slow diffusion which contains the orbits with $\sigma < 10^{-6}$. For $a < 45$ AU, this region is limited to very small spots with low eccentricity. Due to the possible presence of long time scale chaos (Holman and Wisdom,1993; Levison and Duncan, 1993) generated by secular resonances (not detected in this study), the stability of these orbits should need further studies. For semi major axis greater than 45 AU, the secular frequencies of the massless body are too small (at least outside resonances) to lead to low order secular resonances with the giant planets. For this reason, we can suppose that these stable regions, having a proper mean motion diffusion rate lower than 10^{-6} or 10^{-7}, are stable for a very long time.

4. Frequency map for the planetary three-body problem

The method presented here being very general, only small modifications were needed to adapt it to the study of the dynamics of a planetary

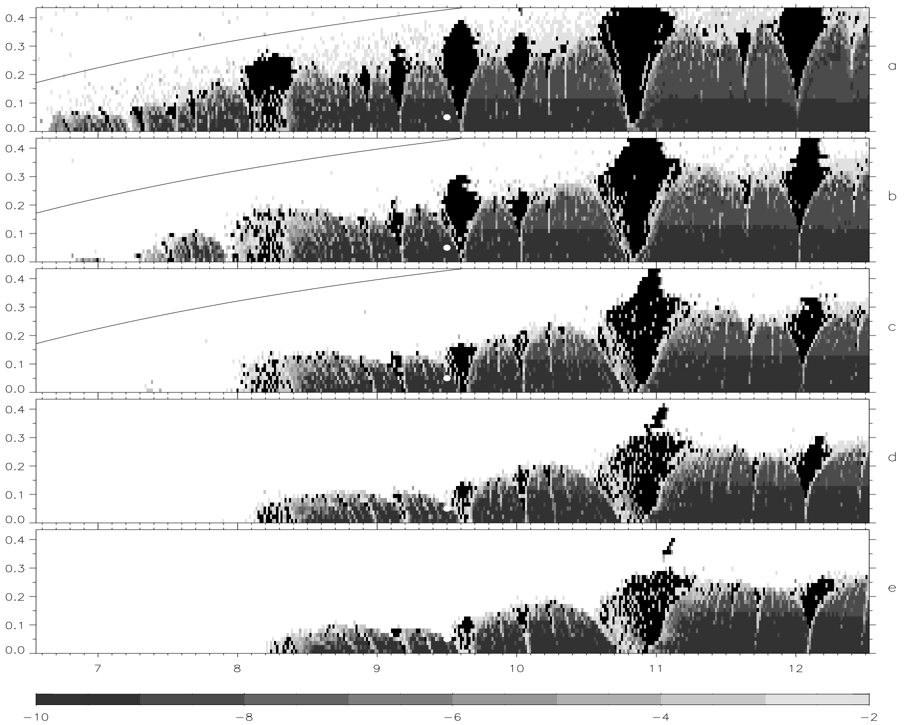

Figure 2. Frequency map of a Sun-Jupiter-Saturn like planetary system, with the present mass values (a), and planetary masses multiplied by a factor 2.5 (b) , 5 (c), 7.5 (d) and 10 (e). The white disc represents the actual location of Saturn.

system where all the bodies have non zero mass. In this case, the dimension of the frequency space is much larger than previously and in order to construct a 2-D map, all the initial elliptical elements of the planets will be fixed, except two action-like variables of a selected planet. For the Sun-Jupiter-Saturn problem presented in Fig.2, only the initial semi major axis (X-axis) and the initial eccentricity of Saturn (Y-axis) are variable, while the integration time is 2 Ma.

In Fig.2.a, the masses of the two planets are the actual masses of Jupiter and Saturn. This provides a very clear picture of the stability domain of the Sun-Jupiter-Saturn system. Close to Jupiter, and up to about 7 AU, there exist no stable regions, while beyond the (1:-2) resonance, at 8.4 UA, we have many very large zones of stability. The actual Saturn (marked by a white disc) evolves in one of these regions close to the left edge of the (2:-5) resonance. In the case of this 3-body planetary system, these stability results, valid on a short time, can be extended on a very much longer time scales (of the order of billion years). Indeed, outside the mean motion resonances, the secular problem provides a good approximation of the long time dynamics. As in this case, the secular problem has only two degrees of freedom, the possible chaotic trajectories are confined between invariant tori, and then induced only local and very small destabilizing effect. An extensive study of the dynamics of this secular problem was done in (Robutel, 1993) for the Jupiter-Saturn like planetary systems, where it was shown that only a few very small chaotic regions exists. A conclusion from this previous study was the following: either a two planet system is destabilized in a short time scale (due to chaotic behavior resulting from mean motion resonances overlap) or it remains stable over a very long a period of time, eventually comparable to the age of the solar system. Indeed, the results presented in Fig.2 describe precisely the zones of stability where the mean motion resonances have no practical effect. These zones can thus be considered also as zones of very long time stability. This simple study provides then important information for the long time stability of these systems.

The four next figures (Fig.2.b-e) correspond to the same experiment except for the planetary masses which are magnified by a factor 2.5 (b), 5 (c), 7.5 (d) and 10 (e). This stresses the destabilizing effect of the resonances. Particularly, we observe gaps on the edges of the libration area (in black) of the main resonances ((1:-2), (2:-5) and (1:-3)) which becomes wider as the perturbation increases. This gives also a precise estimate of the shape and of the size of the stability domain with respect to the perturbation size. This aspect will be more developed in some further study.

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