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Different tidal torques on a planet with a dense atmosphere and consequences to the spin dynamics

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Abstract. Planets with a dense atmosphere present traditional gravitational and thermal tides, but also combined effects resulting from these two. Here we give a complete description of these effects and evaluate their relative contributions to the secular evolution of the planet's spin. In particular, we discuss the importance of determining the exact transfer mechanism of angular momentum from the atmosphere to the surface. It is shown that alternative atmospheric tides may play an important role if this transfer rate is not very effective, although this does not seem to be the present case on Earth or Venus.

1. Introduction

The occurrence, on most open ocean coasts, of high sea tide at about the time of Moon's passage across the meridian, early prompted the idea that our satellite exerts an attraction on the water, although the occurrence of a second high tide when the Moon is on the opposite meridian was a great puzzle. The correct explanation of these tides was first indicated by *Newton* in "*Philosophiæ Naturalis Principia Mathematica*". They are a consequence of the lunar and solar gravitational forces acting in accordance with laws of mechanics. In his work *Newton* realized that the tidal forces also must affect the atmosphere, but he assumed that the atmospheric tides would be too small to be detected, because weather changements would introduce large irregular variations upon barometric measurements.

However, the semi-diurnal oscillations of the atmospheric surface pressure has proven to be one of the most regular of all meteorological phenomena. It is readily detectable by harmonic analysis at any station over the world [eg. *Chapman and Lindzen*, 1970]. The only difference in respect to ocean tides is that they essentially follow the Sun and not the Moon. The reason is that the atmosphere is thermally excited by the Solar heat. Even though tides of gravitational origin are present in the atmosphere, the thermal tides are more important as the pressure variations on the ground are more sensitive to the temperature gradients than to the gravitational ones.

Thermal atmospheric tides regained a special interest with the discovery of the spin of Venus, which rotates very slowly on its axis in a retrograde direction [eg. *Carpenter*, 1964]. In order to explain this peculiar observation, *Gold and Soter* [1969] proposed that this was the result from a balance between gravitational tides, which drives the planet toward synchronous rotation, and thermally driven atmospheric tides, which drives it away.

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Gold and Soter [1971] suggested that the solid part of Venus is perfectly coupled with the atmosphere by surface friction, so that the thermal tides torque is completely transmitted to the planet's spin. This presumes that the atmosphere reached a steady equilibrium (otherwise its rotation would soon accelerate or decelerate until friction with the surface restored the balance of torques), but also requires that the thermal tides lead the Sun (which is not necessarily true) and that there are no other torques on the atmosphere.

The angular momentum transfer from the atmosphere to the solid body is still not well understood, and it is possible that the thermal tides torque are counterbalanced by some other torque on the atmosphere. Stating that "it is impossible to rotate a piece of soap that is perfectly spherical in shape and perfectly slippery", *Hinderer et al.* [1987] try to give an alternative interpretation of the influence of the torque upon atmospheric masses using different mechanisms. Actually, the atmosphere pressure upon the surface gives rise to a deformation of the planet, a pressure bulge, that will also be affected by the solar torque. At the same time, the atmosphere itself exerts a torque over the planet's bulges (gravitational and pressure bulge).

In the present study we try to clarify all tidal effects present for a planet with a dense atmosphere in order to evaluate which mechanisms are the most important for the spin variations. In section two, we revisit in a general frame work the tidal effects theory and the computations of *Hinderer et al.* [1987]. This allows us to evaluate the consequences of each effect as well as their relative strengths (section three).

2. Tidal effects

Tidal effects arise from differential and inelastic deformation of the planet by a perturbing body. Among these effects we count the gravitational tides, thermal atmospheric tides generated from the solar heating of the atmosphere and combined effects of these two (atmospheric tides of gravitational origin are included in gravitational tides).

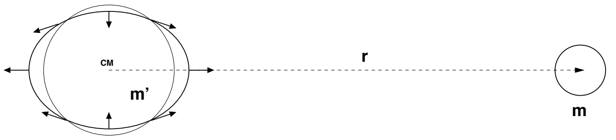

Figure 1. Gravitational tides. The difference between the gravitational force exerted by the mass m on a point of the surface and the center of mass is schematized by the arrows. The planet will deform following the equipotential of all present forces.

The estimations for the contributions to the spin variations are based on a very general formulation of the tidal potential, initiated by *George H. Darwin* [1880]. We first write the complete tidal potential, U^τ, expressed in four angles that characterize the spin: ω, the rotation rate; ε, the obliquity; ψ the general precession angle; and ℓ, the hour angle between the equinox of the date and a fixed point of the equator. Then, the secular contributions to the spin are easily obtained as [eg. *Néron de Surgy and Laskar*, 1997]:

$$\frac{d\omega}{dt} = -m\frac{\partial W^\tau}{\partial l} \, , \quad \frac{dx}{dt} = \frac{m}{\omega}\left(\frac{\partial W^\tau}{\partial \psi} + x\frac{\partial W^\tau}{\partial l}\right) , \quad (1)$$

where $x = \cos\varepsilon$, $W^\tau = U^\tau/C$ and C is the moment of greatest inertia. m is the mass of the interacting body which can be the Sun, a satellite (like the Moon in the case of the Earth) or any particle in the atmosphere.

As we are interested in long term contributions to the spin, we average (1) over the mean anomaly, the longitude of node and perigee of the perturbing body (and of the interacting one when it is not the same). All this work is done with the help of the algebraic manipulator TRIP [*Laskar*, 1989], which expands the potential in Fourier series as done by *Kaula* [1964].

2.1. Gravitational tides

Gravitational tides are raised on one body by another because of the effect of the gravitational gradient across the body. The force experienced by the side facing the perturbing body is stronger than that experienced by the far side. These tides are mainly important upon the solid (or liquid) part of the planet, and are independent of the existence of an atmosphere.

Since the bodies in the Solar System are not perfectly rigid, there will be a distortion that gives rise to a tidal bulge as shown in figure 1. This redistribution of mass modifies the gravitational potential generated by the planet in any point of the space. The additional amount of potential, the tidal potential U^g, is responsible for the modifications in the planet's spin (and orbit). It is given by [eg. *Melchior*, 1971]:

$$U^g = -k_2 \frac{Gm_\odot}{R}\left(\frac{R}{r_\odot}\right)^3\left(\frac{R}{r}\right)^3 P_2(\cos S) \, , \quad (2)$$

where $m_\odot$ is the mass of the perturbing body (here supposed to be the Sun), r and $r_\odot$ respectively the distance from the center of mass of a generic point and of the Sun, and S the angle between these two directions. G is the gravitational constant, R the mean radius of the planet and P_2 the second order Legendre polynomials. k_2 is the second potential Love number, which is determined in part by the planet's internal density distribution. Supposing that the planet is a Maxwell solid[1], *Darwin* [1908] computes:

$$k_2(\sigma) = k_f\sqrt{\frac{1+\sigma^2\upsilon^2/\mu^2}{1+(\sigma^2\upsilon^2/\mu^2)(1+19\mu/2g\bar\rho R)^2}} \, , \quad (3)$$

where $\bar\rho$ is the mean density of the planet, g the surface gravity and k_f the fluid Love number (pertaining to a perfectly fluid body with the same mass distribution as the actual planet). σ is the tidal frequency, a linear combination of the inertial rotation rate ω, and of the mean orbital motion around the Sun n.

In general, imperfect elasticity delays the planet's response to the perturbation by a time lag $\Delta t^g(\sigma)$ (see Fig. 2). Thus, for a fast rotating planet the phase angle of U^g lags behind that of the perturbing potential by an angle $\delta^g(\sigma)$ such that [*Kaula*, 1964]: $\delta^\tau(\sigma) = \frac{1}{2}\sigma\Delta t^\tau(\sigma)$

Now, using (1), we are able to write the contributions to the spin (when the Sun is also the interacting body, $m = m_\odot$):

$$\frac{d\omega}{dt} = -\frac{Gm_\odot^2R^5}{Ca^6}\sum_\sigma b^g(\sigma)\Lambda_\sigma^g(x,e) \, , \quad (4)$$

$$\frac{dx}{dt} = -\frac{Gm_\odot^2R^5}{Ca^6}\frac{x^2-1}{\omega}\sum_\sigma b^g(\sigma)\Theta_\sigma^g(x,e) \, , \quad (5)$$

where a is the semi major axis and e the eccentricity of the planet's orbit. When the eccentricity is small, we can neglect the terms in e^2, we have then:

$$\begin{aligned} \textstyle\sum_\sigma b^\tau(\sigma)\Lambda_\sigma &= b^\tau(\omega)\tfrac{3}{4}x^2\left(1-x^2\right) \\ &+ b^\tau(\omega-2n)\tfrac{3}{16}\left(1+x\right)^2\left(1-x^2\right) \\ &+ b^\tau(\omega+2n)\tfrac{3}{16}\left(1-x\right)^2\left(1-x^2\right) \\ &+ b^\tau(2\omega)\tfrac{3}{8}\left(1-x^2\right)^2 \\ &+ b^\tau(2\omega-2n)\tfrac{3}{32}\left(1+x\right)^4 \\ &+ b^\tau(2\omega+2n)\tfrac{3}{32}\left(1-x\right)^4 \, , \end{aligned} \quad (6)$$

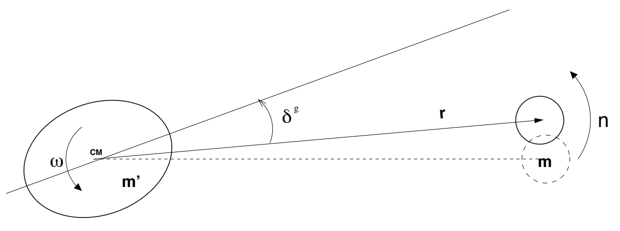

Figure 2. Phase lag for gravitational tides. The tidal deformation takes a delay time Δt^g to attain the equilibrium. During this time, the planet turns by an angle $\omega\Delta t^g$ and the Sun by $n\Delta t^g$. For $\varepsilon = 0$, the bulge phase lag is given by $\delta^g \simeq (\omega - n)\Delta t^g$.

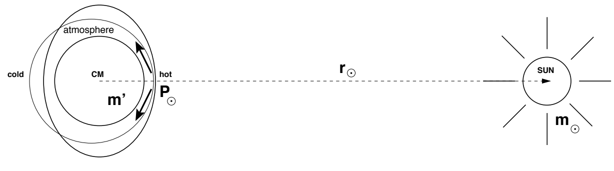

Figure 3. Thermal atmospheric tides. The atmosphere's heating decreases with the distance to the sub-solar point $P_\odot$. The atmospheric mass redistribution is essentially decomposed in a weak daily tide (round shape) and in a strong half-daily tide (oval shape).

and

$$\begin{aligned}\textstyle\sum_\sigma b^\tau(\sigma)\Theta_\sigma &= b^\tau(2n)\tfrac{9}{16}\left(1-x^2\right)\\ &+ b^\tau(\omega)\tfrac{3}{4}x^3\\ &- b^\tau(\omega-2n)\tfrac{3}{16}(1+x)^2(2-x)\\ &+ b^\tau(\omega+2n)\tfrac{3}{16}(1-x)^2(2+x)\\ &+ b^\tau(2\omega)\tfrac{3}{8}x\left(1-x^2\right)\\ &- b^\tau(2\omega-2n)\tfrac{3}{32}(1+x)^3\\ &+ b^\tau(2\omega+2n)\tfrac{3}{32}(1-x)^3 .\end{aligned} \quad (7)$$

For gravitational tides, the factor $b^\tau(\sigma)$ is given by:

$$b^g(\sigma) = k_2(\sigma)\sin 2\delta^g(\sigma) = k_2(\sigma)\sin\left(\sigma\Delta t^g(\sigma)\right) . \quad (8)$$

Dissipation of the mechanical energy of tides in the planet's interior is responsible for the time delay $\Delta t^g(\sigma)$ between the position of "maximal tide" and the sub-solar point. A commonly used dimensionless measure of tidal damping is the quality factor Q, defined as the inverse of the "specific" dissipation and related to the phase lags by [*Munk and MacDonald*, 1960]:

$$Q_\sigma = \frac{2\pi E}{\Delta E} = \cot 2\delta(\sigma) , \quad (9)$$

where E is the total tidal energy stored in the planet and ΔE the energy dissipated per cycle. As the rheology of terrestrial planets is badly known, the relation between the frequency and the time lag is often subjected to some rough approximations.

2.2. Thermal atmospheric tides

The differential absorption of the Solar heat by the planet's atmosphere gives rise to local variations of temperature and consequently to pressure gradients. The mass of the atmosphere is then permanently redistributed, adjusting for an equilibrium position. More precisely, the particles of the atmosphere move from the high temperature zone (at the sub-solar point) to the low temperature areas. Indeed, observations on Earth show that the pressure redistribution is essentially a superposition of two pressure waves as shown in figure 3: a diurnal tide of small amplitude and a strong semi-diurnal tide [see *Chapman and Lindzen*, 1970].

As for gravitational tides, the redistribution of mass in the atmosphere gives rise to an atmospheric bulge that modifies the gravitational potential generated by the atmosphere in any point of the space. The tidal potential U^a responsible for the spin changes is obtained in appendix (Eq. A2) subtracting the term of constant pressure ($l = 0$) and also eliminating the diurnal terms ($l = 1$) that correspond to a displacement of the center of mass of the atmosphere bulge with no dynamical implications. Since $r \gg R$ we can retain only the semi-diurnal terms ($l = 2$), so:

$$U^a = -\frac{3}{5}\frac{\tilde{p}_2}{\bar{\rho}}\left(\frac{R}{r}\right)^3 P_2(\cos S) , \quad (10)$$

where $\tilde{p}_2$ is the second order surface pressure variations (see expression A3). Here, too, there is a delay $\Delta t^a(\sigma)$ before the response of the atmosphere to the excitation (see Fig. 4).

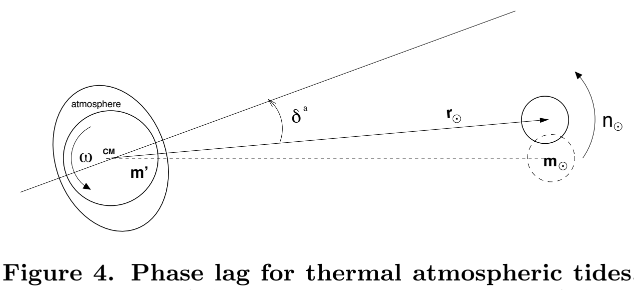

Figure 4. Phase lag for thermal atmospheric tides. During the time Δt^a the planet turns by an angle $\omega\Delta t^a$ and the Sun by $n\Delta t^a$. For $\varepsilon = 0$, the bulge phase lag is given by $\delta^a \simeq (\omega - n)\Delta t^a$.

According to expression (1), the dynamical equations are then (here $m = m_\odot$):

$$\frac{d\omega}{dt} = -\varsigma\frac{3m_\odot R^3}{5C\bar{\rho}a^3}\sum_\sigma b^a(\sigma)\Lambda^a_\sigma(x,e) , \quad (11)$$

$$\frac{dx}{dt} = -\varsigma\frac{3m_\odot R^3}{5C\bar{\rho}a^3}\frac{x^2-1}{\omega}\sum_\sigma b^a(\sigma)\Theta^a_\sigma(x,e) , \quad (12)$$

where ς represents the fraction of angular momentum communicated to the solid body's spin. The expressions of Λ^a_σ and Θ^a_σ are different from their analogs in gravitational tides, Λ^g_σ and Θ^g_σ. Nevertheless, when neglecting the terms in e^2, they become equal:

$$\Lambda^a_\sigma(e=0) = \Lambda^g_\sigma(e=0) = \Lambda_\sigma , \quad (13)$$

$$\Theta^a_\sigma(e=0) = \Theta^g_\sigma(e=0) = \Theta_\sigma . \quad (14)$$

This allows us to use for atmospheric tides the same expressions (6) and (7) where $b^\tau(\sigma)$ is now:

$$b^a(\sigma) = \tilde{p}_2(\sigma)\sin 2\delta^a(\sigma) = \tilde{p}_2(\sigma)\sin(\sigma\Delta t^a(\sigma)) . \quad (15)$$

Siebert [1961] and *Chapman and Lindzen* [1970] showed that the amplitudes of the pressure variations on the ground can be given by:

$$\tilde{p}_2(\sigma) = \mathrm{i}\frac{\gamma}{\sigma}\tilde{p}_0\left(\nabla\cdot\vec{v}_\sigma - \frac{\gamma-1}{\gamma}\frac{J_\sigma}{gH_0}\right) , \quad (16)$$

where $\gamma = 7/5$ for a perfect gas, $\vec{v}$ is the velocity of tidal winds, J the amount of heat absorbed or emitted by a unit mass of air per unit time and H_0 is the scale height at the surface. The imaginary number above causes the pressure variations to lead the Sun as long as the phase lag $\delta^a(\sigma) < 90°$ [*Chapman and Lindzen*, 1970, *Dobrovolskis and Ingersoll*, 1980, *Correia et al.*, 2003]. Indeed, replacing (16) in (15) we have

$$\begin{aligned} b^a(\sigma) &= |\tilde{p}_2(\sigma)|\sin 2\left(\delta^a(\sigma)\pm\pi/2\right) = \\ &= -|\tilde{p}_2(\sigma)|\sin 2\delta^a(\sigma) .\end{aligned} \quad (17)$$

2.3. Gravitational torque upon pressure bulge

When a dense atmosphere is present, its mass and the differential pressure that it exerts upon the surface also distorts the planet giving rise to what we call a "pressure" bulge [*Hinderer et al.*, 1987]. This bulge is similar to the tidal one except that the potential that raises the deformation is no longer the gravitational potential of a perturbing body, but the potential generated by the entire atmosphere (A2). The Love number that accounts for the atmospheric deformation of the surface is the load Love number k_2' [eg. *Lambeck*, 1988]. The additional potential generated by the pressure bulge is then given by:

$$U^{gp} = k_2' U^a , \quad (18)$$

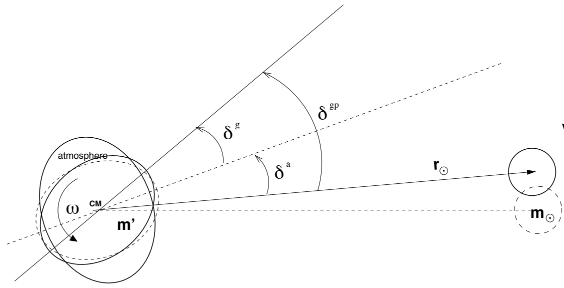

Figure 5. Phase lag of the pressure bulge. First, the atmosphere takes a time Δt^a to respond to the Sun heating, then the planet takes a time Δt^g to respond to the effect of the atmosphere upon the surface. For $\varepsilon = 0$, the bulge phase lag is given by $\delta^{gp} = \delta^a + \delta^g \simeq (\omega - n) \times (\Delta t^a + \Delta t^g)$.

where U^a is the same as in expression (10). *Molodenskiy* [1977] showed that $k_2' = k_2 - h_2$, thus k_2' is always negative and we therefore rewrite the above expression as:

$$U^{gp} = |k_2'| \, \frac{3}{5} \frac{\tilde{p}_2}{\bar{\rho}} \left(\frac{R}{r} \right)^3 P_2(\cos S) \, . \tag{19}$$

When the interacting body is the Sun, the dynamical equations (1) become then ($m = m_\odot$),

$$\frac{d\omega}{dt} = -\frac{3 m_\odot R^3}{5 C \bar{\rho} a^3} \sum_\sigma b^{gp}(\sigma) \, \Lambda^a_\sigma(x, e) \, , \tag{20}$$

$$\frac{dx}{dt} = -\frac{3 m_\odot R^3}{5 C \bar{\rho} a^3} \frac{x^2 - 1}{\omega} \sum_\sigma b^{gp}(\sigma) \, \Theta^a_\sigma(x, e) \, . \tag{21}$$

The time lag here is given by $\Delta t^{gp}(\sigma) = \Delta t^a(\sigma) + \Delta t^g(\sigma)$, since the response of the surface to the atmosphere perturbation only takes place after the deformation of the atmosphere (Fig. 5). Hence,

$$\begin{aligned} b^{gp}(\sigma) &= k_2' \tilde{p}_2(\sigma) \sin 2\delta^{gp} \\ &= |k_2' \, \tilde{p}_2(\sigma)| \sin 2(\delta^a + \delta^g) \, . \end{aligned} \tag{22}$$

2.4. Pressure torque upon tidal bulge

The only difference between this effect and the gravitational tides is that the interacting body is no longer the Sun. Now, it is the entire atmosphere and so, equations (1) must be written as a sum of every particle in the atmosphere. However, contrarily to the Sun, each particle has a double effect upon the surface: it pulls it like the Sun did, but it also pushes it by pressure. In order to evaluate this double behavior, we introduce an "equivalent" mass, $\mathcal{M}^{eq}$, that acts like a simple mass, but that takes into account both effects (see appendix A):

$$\mathcal{M}^{eq}_l = \frac{2 - 2l}{3} \mathcal{M} \, . \tag{23}$$

The "equivalent" mass depends on each harmonic l. For $l = 2$, we have $\mathcal{M}^{eq}_2 = -\frac{2}{3}\mathcal{M}$ and thus from (1), the contributions to dynamical equations are:

$$\begin{aligned} \frac{d\omega}{dt} &= -\frac{\partial}{\partial l} \int W^g \, d\mathcal{M}^{eq}_2 = \frac{2}{3} \frac{\partial}{\partial l} \int W^g \, d\mathcal{M} = \\ &= \frac{2 m_\odot R^3}{5 C \bar{\rho} a^3} \sum_\sigma b^{pg}(\sigma) \, \Lambda^a_\sigma(x, e) \, , \end{aligned} \tag{24}$$

$$\begin{aligned} \frac{dx}{dt} &= -\frac{2}{3\omega} \left(\frac{\partial}{\partial \psi} + x \frac{\partial}{\partial \ell} \right) \int W^g \, d\mathcal{M} = \\ &= \frac{2 m_\odot R^3}{5 C \bar{\rho} a^3} \frac{x^2 - 1}{\omega} \sum_\sigma b^{pg}(\sigma) \, \Theta^a_\sigma(x, e) \, . \end{aligned} \tag{25}$$

Since in this case the responses of both the solid body and the atmosphere are independent, the time lag is $\Delta t^g(\sigma) - \Delta t^a(\sigma)$, as shown in figure 6. This has an important consequence, as the sign of $\delta^{pg}(\sigma)$ determines whether this effect accelerates the planet or brakes its rotation.

$$\begin{aligned} b^{pg}(\sigma) &= k_2 \tilde{p}_2(\sigma) \sin 2\delta^{pg} = \\ &= k_2 |\tilde{p}_2(\sigma)| \sin 2(\delta^a - \delta^g) \, . \end{aligned} \tag{26}$$

2.5. Pressure torque upon pressure bulge

Here the difference from the preceding case resides in the potential that generates the bulge. Therefore, we only need to replace U^g by U^{gp} in the dynamical equations (1):

$$\frac{d\omega}{dt} = -\frac{\partial}{\partial l} \int W^{gp} \, d\mathcal{M}^{eq}_2 = \frac{2}{3} \frac{\partial}{\partial l} \int W^{gp} \, d\mathcal{M} =$$

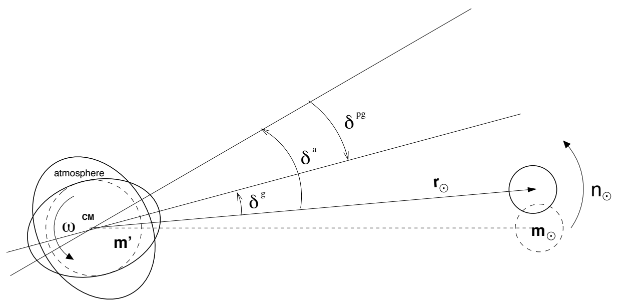

Figure 6. Pressure torque upon tidal bulge. The response of the atmosphere to the solar heating Δt^a and the solid deformation of the planet by a perturbing body Δt^g are independent. The time lag depends then on the difference between these two effects. For $\varepsilon = 0$, the phase lag is given by $\delta^{pg} = \delta^g - \delta^a \simeq -(\omega - n) \times (\Delta t^a - \Delta t^g)$.

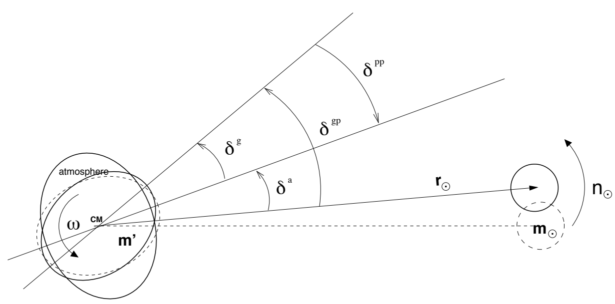

Figure 7. Pressure torque upon pressure bulge. The atmosphere response to the solar heating Δt^a and the deformation of the planet by the atmosphere Δt^{gp} are independent. The time lag depends then on the difference between these two effects. For $\varepsilon = 0$, the phase lag is given by $\delta^{pp} = \delta^{gp} - \delta^a \simeq (\omega - n) \times \Delta t^g$.

$$= \frac{6R}{25GC\bar{\rho}^2} \sum_\sigma b^{pp}(\sigma)\, \Lambda_\sigma^{pp}(x,e)\,, \quad (27)$$

$$\frac{dx}{dt} = -\frac{2}{3\omega}\left(\frac{\partial}{\partial\psi} + x\frac{\partial}{\partial\ell}\right)\int W^{gp}\, d\mathcal{M} =$$

$$= \frac{6R}{25GC\bar{\rho}^2}\frac{x^2-1}{\omega}\sum_\sigma b^{pp}(\sigma)\,\Theta_\sigma^{pp}(x,e)\,. \quad (28)$$

The expressions of Λ_σ^{pp} and Θ_σ^{pp} are different from their analogous for the gravitational and atmospheric tides. However, as for the other tidal effects, when neglecting the eccentricity terms in e^2, they become equal, which allow us to use the truncated series (6) and (7):

$$\Lambda_\sigma^{pp}(e=0) = \Lambda_\sigma\,, \quad \Theta_\sigma^{pp}(e=0) = \Theta_\sigma\,. \quad (29)$$

The time lag here is a combination of the last two cases (see Fig. 7):

$$(\Delta t^a + \Delta t^g) - \Delta t^a = \Delta t^g\,, \quad (30)$$

and so,

$$b^{pp}(\sigma) = k_2'\tilde{p}_2\tilde{p}_2 \sin 2\delta^{pp} = -|k_2'|\,|\tilde{p}_2(\sigma)|^2 \sin 2\delta^g\,. \quad (31)$$

3. Secular evolution of the spin

Tidal torques are usually very small when compared with precessional torques or some other gravitational perturbations from the planets. However, these torques are dissipative and their continuous effect upon the planet give rise to secular changes of the spin over long periods of time. For instance, the effect of the Moon gravitational tides on our planet increases the Earth's length of the day by about 2 ms each century [eg. *Stephenson and Morrison*, 1984]. On Venus, atmospheric tides are essential to explain the present observed retrograde rotation [eg. *Correia and Laskar*, 2001]. It is thus important to evaluate the contribution of each tidal effect in order to understand the possible modifications of the spin since the formation of the Solar System, about 4.6 Gyr ago.

In our work we have considered several kinds of tidal effects: gravitational tides (g), thermal atmospheric tides (a) and combinations of these two kinds, namely, gravitational torque upon pressure bulge (gp), pressure torque upon tidal bulge (pg) and pressure torque upon pressure bulge (pp). Although they have the same nature (a periodic response of the planet to a solar perturbation), their contributions to the dynamical equations are different. Atmospheric tides for instance, can either control the spin evolution or simply be neglected, according to the tidal frequency.

As the tidal effects' contributions to the dynamical equations are the same to first order in e, only the constant K^τ appearing in expressions (4), (11), (20), (24) and (27) and the respective dissipation factor $b^\tau(\sigma)$ change. For each tidal frequency, the magnitude of the effect then depends on the product of the constant K^τ and $b^\tau(\sigma)$. In table 1, we reported the different tides and their relative strength. The magnitude computations are performed for the planet Venus (where atmospheric tides are very important) using the data from *Yoder* [1995], the "heating at the ground" model for surface pressure variations (Eq. 32) [*Dobrovolskis and Ingersoll*, 1980] and the ratio $\Delta t^a/\Delta t^g = 36.5$ [*Correia et al.*, 2003]. All magnitudes are given as a function of the tidal frequency and divided by the magnitude of gravitational tides.

Table 1. Tidal effects comparison. The tidal effects' contributions to the dynamical equations are the same to first order in e, only the constant K^τ and the dissipation factor $b^\tau(\sigma)$ change for each tidal effect (τ) and frequency (σ).

Tide (τ)	K^τ	$b^\tau(\sigma)$	Magn.
g	$-\frac{Gm_\odot^2 R^5}{Ca^6}$	$k_2 \sin 2\delta^g$	1
a	$-\varsigma\frac{3m_\odot R^3}{5C\bar{\rho}a^3}$	$-\lvert\tilde{p}_2\rvert \sin 2\delta^a$	$1.92\,\varsigma\left(\frac{2n}{\sigma}\right)$
gp	$-\frac{3m_\odot R^3}{5C\bar{\rho}a^3}$	$\lvert k_2'\,\tilde{p}_2\rvert \sin 2(\delta^a+\delta^g)$	$0.40\left(\frac{2n}{\sigma}\right)$
pg	$\frac{2m_\odot R^3}{5C\bar{\rho}a^3}$	$k_2\lvert\tilde{p}_2\rvert \sin 2(\delta^a-\delta^g)$	$0.32\left(\frac{2n}{\sigma}\right)$
pp	$\frac{6R}{25GC\bar{\rho}^2}$	$-\lvert k_2'\rvert\,\lvert\tilde{p}_2\rvert^2 \sin 2\delta^g$	$10^{-4}\left(\frac{2n}{\sigma}\right)^2$

3.1. Gravitational tides

The secular effect of gravitational tides over the spin is known since the first studies on the subject, as it is quite independent of the dissipation models adopted [eg. *Munk and MacDonald*, 1960]. In fact, the only difficulty is related to the quality factor Q (see expression 9), whose variations do not change the general look of the dynamical equations (4) and (5). Thus, it has been shown that gravitational tides always slows down the planet as long as the rotation rate is faster than the synchronous rotation. For a low orbital eccentricity, this will be the synchronous resonance, which corresponds to the equilibrium spin rate for this kind of tides. Concerning the obliquity, the axis of the planet may cross intermediate states, but at the end, when the synchronous resonance is achieved, gravitational tides set the obliquity at near zero degrees.

3.2. Atmospheric tides

The analysis of the consequences of atmospheric tides to the spin is less straightforward than for gravitational tides. Indeed, the presence of the surface pressure variations in dynamical equations prevents a general behavior, since the coefficient $\tilde{p}_2$ depends on the atmosphere composition, heating and dynamics (see expression 16). We are then compelled to consider specific models to account for all the unknown dissipation parameters.

For a planet like Venus, with a dense atmosphere, atmospheric tides become very important. Several studies have been performed to model the surface pressure variations on Venus. One simple model commonly used that seems to be in a good agreement with the observations is the "heating at the ground" model [*Dobrovolskis and Ingersoll*, 1980]. In this model we suppose that all the solar flux absorbed by the ground F_s is immediately deposited in a thin layer of atmosphere at the surface. Here, the heating distributing may be written as a delta-function just above the ground, and neglecting $\vec{v}$ over the thin heated layer, expression (16) simplifies[2]:

$$\tilde{p}_2(\sigma) = \mathrm{i}\,\frac{5}{16}\,\frac{\gamma-1}{\sigma}\,\frac{F_s}{H_0} \;\propto\; \sigma^{-1}\,. \quad (32)$$

This model works for Venus, because tides in the upper atmosphere are decoupled from the ground by the disparity between their rotation rates.

3.2.1. Rotation rate variations.

All the conclusions derived here and in the next section will be restricted to $\omega > 0$, but results for ω positive always can be extended to negative since the couple $(-|\omega|, \varepsilon)$ behaves identically to the couple $(|\omega|, \varepsilon + \pi)$.

In a fast rotation stage ($\omega > 2n$), where the planet is believed to spend most of its evolution, the evolution tendency of $d\omega/dt$ is the same for any dissipation model, because all the terms in (6) have the same sign. Thus, the only consequence of tidal effects to the rotation rate is either to accelerate it or to brake it, depending on the sign of the product between $b^\tau(\sigma)$ and the respective constant K^τ displayed in table 1. For $\delta^a(\sigma) < 90°$, thermal atmospheric tides (a) and the pressure torque upon tidal bulge (pg) accelerate the planet, while the gravitational torque upon pressure bulge (gp) and the pressure torque upon pressure bulge (pp) decelerate it.

On the contrary, in the slow rotation regime ($0 < \omega < 2n$) some terms in (6) become negative and we must considerer the various dissipation models. As the function Λ_σ (6) is a polynomial of degree four in x, we still know that there exists for each rotation rate at most four roots of $d\omega/dt$, two corresponding to stable points and the other two to unstable points of the rotation rate.

3.2.2. Obliquity variations.

We cannot dissociate the evolution of the rotation rate from the obliquity evolution: a modification of ε gives rise to a variation on ω and vice versa. In the expression of the obliquity variations for tidal effects (7), different sign terms are present for any rotation rate, so the obliquity variations will always be model dependent.

The obliquity variations depend on the surface pressure variations amplitude, $\tilde{p}_2(\sigma)$, which are inversely proportional to σ (16). Thus, for fast rotation rates ($\omega \gg n$), dx/dt is dominated by the first term in (7). There will be two

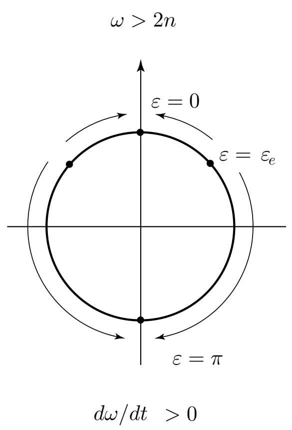

Figure 8. General consequences of the thermal atmospheric tides for $\omega > 2n$ and $\delta^a(\sigma) < 90°$. The rotation rate is always accelerated and the obliquity is essentially reversed. However, for slow rotation rates the spin axis will be put right if the obliquity is inferior to a critical value $\cos\varepsilon_e \simeq -1 + \frac{3\omega}{4n} - \frac{1}{4}(9\frac{\omega^2}{n^2} - 48\frac{\omega}{n} + 96)^{1/2}$. For the pressure torque on the tidal bulge the schema is the same. For the other combined tides stable points become unstable and vice versa (we change the direction of the arrows).

obliquity values where dx/dt vanishes, $0°$ ($x = 1$) and $180°$ ($x = -1$), one stable and the other unstable (the stability depends on the sign of the product of $b^\tau(\sigma)$ with the respective constants K^τ of table 1). For the effects which brake the rotation, the stability is reached at $\varepsilon = 0°$ (whereas to the effects which accelerate the planet it is reached at $\varepsilon = 180°$). As we approach slow rotation rates ($\omega \sim n$), all the terms of (7) become important and more complex behaviors are expected. Nevertheless, since the function $\Theta_\sigma(x)$ appearing in expression (7) is a polynomial of degree three in x, then dx/dt has at most three zeros in the interval $]-1, 1[$. These roots can be computed analytically, but this request the use of a specific dissipation model. Using the "heating at the ground" model (see expression 32), we observe that for $\omega > 2n$, the stability (or instability) point at $\varepsilon = 0°$ will be displaced to an equilibrium obliquity close to ε_e such that:

$$\cos\varepsilon_e \simeq -1 + \frac{3\omega}{4n} - \frac{1}{4}\sqrt{9\frac{\omega^2}{n^2} - 48\frac{\omega}{n} + 96}\,. \qquad (33)$$

The point $\varepsilon = 0°$ is still an equilibrium point, but becomes now unstable for the effects which brake the rotation, and stable for the effects which accelerate the planet. For the thermal atmospheric tides, we have schematized the main behavior in figure 8. When ($0 < \omega < 2n$) there are inversions in the signs of the terms $b^\tau(\sigma)$ and a general solution becomes much more complicated.

4. Relative strength of tides

All atmospheric tidal terms $b^\tau(\sigma)$ are proportional to the factor $2n/\sigma$, resulting from the surface pressure variations term $\tilde{p}_2$ (see table 1). Then, for very low values of σ, their strength become more important than gravitational tides. On the other hand, for initial rotation rates we typically have $\sigma \gg n$ and thus the contribution of atmospheric tides to the rotation rate can be neglected. This simple result is important as we may not know exactly when a dense atmosphere was formed during the planet's evolution [eg. *Pepin*, 1991]. If we assume a fast original rotation period, the formation of an atmosphere in the very beginning of the Solar System or only after some hundreds of million years will not directly affect the evolution of the rotation rate (see for the case of Venus *Correia and Laskar* [2003]). However, the same is not true for the obliquity evolution. Due to the presence of the term $b^\tau(2n)\frac{9}{16}(1 - x^2)$ (first term in expression 7) there is a permanent small contribution of atmospheric tides for obliquities around $90°$.

4.1. Pressure torque upon pressure bulge

Looking in detail to each atmospheric tidal effect, we see that tides resulting from the pressure torque upon pressure bulge (pp) only become of the same order of magnitude of the other atmospheric tides for $|\sigma| < n/100$ (see table 1). For these values of the tidal frequency the hypothesis made to derive the surface pressure variations (16), is no longer valid [*Siebert*, 1961]. Thus, it is better to take $\tilde{p}_2 = 0$, which corresponds to the *solar equilibrium tide*, the real value of the surface pressure variations for $\sigma = 0$ [*Chapman and Lindzen*, 1970]. Therefore, we can ignore the contribution of this kind of tides to the dynamical evolution.

4.2. Remaining atmospheric tides

In respect to the remaining atmospheric tides (a), (gp) and (pg), we can simplify the motion equations if we make the assumption that the difference between the dissipation time lags are of the same order as for the Earth and Venus. For gravitational tides, we estimate the time lag $\Delta t^g = 638$ s [*Mignard*, 1979, *Néron de Surgy and Laskar*, 1997], while for the atmospheric time lag we use $\Delta t^a = 13\,428\,s$ [*Volland*, 1988] which gives $\Delta t^a \simeq 21 \Delta t^g$. For Venus, the present equilibrium between gravitational and atmospheric tides gives $\Delta t^a \simeq 37 \Delta t^g$ [*Correia et al.*, 2003]. Since $2\delta^\tau = \sigma \Delta t^\tau$, we can neglect δ^g_σ with respect to δ^a_σ:

$$\begin{aligned} \sin 2(\delta^a \pm \delta^g) &= \sin \left[\sigma \Delta t^a \left(1 \pm \frac{\Delta t^g}{\Delta t^a} \right) \right] \simeq \\ &\simeq \sin \sigma \Delta t^a = \sin 2\delta^a_\sigma \, . \end{aligned} \quad (34)$$

The different atmospheric tidal effects can then be regarded as a single one governed by the system of equations (11) and (12), where ς is replaced by ς':

$$\varsigma' = \varsigma + k'_2 + \frac{2}{3} k_2 = \varsigma + \frac{5}{3} k_2 - h_2 \, . \quad (35)$$

For the Earth, $k_2 \simeq 0.31$ and $h_2 \simeq 0.61$ [*Lambeck*, 1988] which gives $\varsigma' = \varsigma - 0.09$. In the case of Venus we have $k_2 \simeq 0.25$ and $h_2 \simeq 0.46$ [*Yoder*, 1995], so $\varsigma' = \varsigma - 0.04$.

4.3. Discussion

4.3.1. Efficient thermal tides.

If the atmosphere of a planet is tightly coupled with its solid body ($\varsigma = 1$), only the contribution from thermal atmospheric tides needs to be retained since $\varsigma' \simeq 1$ (Eq. 35). In the case of Venus, with the assumption of efficient thermal tides, the unknown phase lag $\delta^a(\sigma)$ must also be smaller than 90°. Indeed, if that was not the case, the present observed equilibrium [*Gold and Soter*, 1969, *Correia and Laskar*, 2001] could not be maintained: atmospheric tides would decrease the rotation rate, and will then be unable to counteract the braking effect from the gravitational tides.

4.3.2. Weak thermal tides.

If the angular momentum transfer rate from thermal tides to the solid planet is inferior of about $\varsigma < 0.1$, according to expression (35) thermal tides become of the same order of magnitude as the other atmospheric tides (gravitational torque upon the pressure bulge and the pressure torque upon the tidal bulge). In this situation, all the different tides must then be taken into account to the secular spin evolution. We can then expect different behaviors for planets with different atmospheres' compositions and internal structures, as the Love numbers k'_2 and k_2 will change (Eq. 35).

4.3.3. Absence of thermal tides.

Assume now, as in the hypothesis of *Hinderer et al.* [1987], that $\varsigma \simeq 0$ (slippery sphere model) and that the present rotation state of Venus could be explained without considering the thermal tides. From expression (35) we then have $\varsigma' < 0$. If we use, as in the example presented at the end of *Hinderer et al.*'s paper, a lag $\delta^a(\sigma)$ smaller than 90°, it is impossible to reach the final observed equilibrium as the effect of the remaining atmospheric tides can only decrease the rotation rate. However, since many of the properties of the venusian atmosphere are unknown, such as the ground pressure variations amplitude and the atmospheric phase lag, we may as well suppose that $\delta^a(\sigma) > 90°$. In this special situation, the consequences of all atmospheric tides are reversed: thermal tides and the pressure torque upon the tidal bulge decrease the rotation rate while the gravitational torque upon the pressure bulge is the only capable of counteracting the braking effect from gravitational tides. Thus, the present observed spin configuration of Venus could be obtained thanks to the gravitational torque upon the pressure bulge (see section 2.3), but, in order to maintain the present observerd equilibrium on Venus, it will require for the the amplitude of the ground pressure variations to be about $1/0.04 = 25$ times larger than for the efficient thermal tides scenario.

5. Conclusion

We have revisited here the theory of tidal effects and analyzed in detail the tides resulting from the presence of a dense atmosphere. For each effect we obtain the contributions to the spin dynamics. Their consequences to the secular evolution of the planet have been studied and we show that the pressure torque upon the pressure bulge can always be neglected.

We stress especially the importance of determining the angular momentum transfer rate from the atmosphere to the solid body. Our study confirms that if the atmosphere is in equilibrium with the surface, only the classical thermal tides play an important role among all atmospheric tides. However, if the transfer of angular momentum from thermal tides is only partial, the effect of the gravitational torque upon the pressure bulge and the pressure torque upon the tidal bulge may become significative and give alternative spin evolutions. In the particular case of Venus, for $\delta^a(\sigma) < 90°$ the effect of thermal atmospheric tides cannot be neglected, since without their contribution the remaining tides can only slow down the rotation, preventing the planet from reaching the present observed equilibrium. For atmospheric phase lags $\delta^a(\sigma) > 90°$ this equilibrium becomes impossible to maintain, unless we use the *Hinderer et al.*'s slippery sphere model. In that case, the equilibrium will become possible thanks to the effect of the gravitational torque upon the pressure bulge become essential, but it will require the ground pressure variations to be about 25 times larger than in the former equilibrium generated by thermal tides. The determination by future observations whether the atmospheric phase lag on Venus is smaller or higher than 90° should allow to solve this question.

Appendix A: The "equivalent" mass

Each particle in the atmosphere has a double effect upon the surface: it pulls it by gravitational attraction, but it also pushes it by pressure. An easy way to model this, it is to introduce an "equivalent" mass, $\mathcal{M}^{eq}$, that behaves like a simple mass, but that takes into account both effects. Here we will show how to obtain the "equivalent" mass using the load Love numbers theory [eg. *Farrell*, 1972].

The gravitational potential generated by all of the particles in the atmosphere at a generic point of the space $\vec{r}$ is given by:

$$V^a(\vec{r}) = -G \int_{(\mathcal{M})} \frac{d\mathcal{M}}{|\vec{r} - \vec{r}_\mathcal{M}|} \, , \quad (A1)$$

where $\vec{r}_\mathcal{M}$ is the position of the atmosphere mass element $d\mathcal{M}$. Making the hypothesis that the planet's radius is constant and that the height of the atmosphere can be neglected when compared with it, we rewrite the previous expression as [*Correia et al.*, 2003]:

$$V^a(\vec{r}) = -\frac{1}{\bar{\rho}} \sum_{l=0}^{+\infty} \frac{3}{2l+1} \tilde{p}_l \left(\frac{R}{r}\right)^{l+1} P_l(\cos S) , \quad \text{(A2)}$$

where P_l are the Legendre polynomials of order l and $\tilde{p}_l$ the coefficients of the expansion of the surface pressure p_s as:

$$p_s(S) = \sum_{l=0}^{+\infty} \tilde{p}_l \, P_l(\cos S) . \quad \text{(A3)}$$

The pressure force per unit mass exerted by the atmosphere upon the surface is given by:

$$\vec{f}^p = -\frac{1}{\bar{\rho}} \nabla p_s = -\sum_{l=0}^{\infty} \frac{\tilde{p}_l}{\bar{\rho}} \nabla P_l(\cos S) , \quad \text{(A4)}$$

whereas, the gravitational force per unit mass is:

$$\begin{aligned} \vec{f}^g &= -\nabla V^a(\vec{r}) = -\sum_{l=0}^{\infty} \nabla V_l^a = \\ &= \sum_{l=0}^{\infty} \left(\frac{3}{2l+1}\right) \frac{\tilde{p}_l}{\bar{\rho}} \nabla P_l(\cos S) . \end{aligned} \quad \text{(A5)}$$

The total force, $\vec{f} = \sum_l \vec{f}_l$, is then given by:

$$\vec{f}_l = \vec{f}_l^g + \vec{f}_l^p = \left(\frac{3}{2l+1} - 1\right) \frac{\tilde{p}_l}{\bar{\rho}} \nabla P_l(\cos S) , \quad \text{(A6)}$$

hence

$$\vec{f}_l = \frac{2-2l}{3} \nabla V_l^a = \frac{2-2l}{3} \vec{f}_l^g , \quad \text{(A7)}$$

that is, for each harmonic l, the total force exerted by the atmosphere upon the surface can be replaced by a gravitational force. Since the gravitational force is proportional to the mass of the particles in the atmosphere $\mathcal{M}$, we can say that for each harmonic the total force is the gravitational force generated by the particles with an "equivalent" mass $\mathcal{M}_l^{eq}$ defined as:

$$\mathcal{M}_l^{eq} = \frac{2-2l}{3} \mathcal{M} . \quad \text{(A8)}$$

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Notes

1. A material is called Maxwell solid when it responds to stresses like a massless, damped harmonic oscillator (except in shear instead of in compression). It is characterized by a rigidity (or shear modulus) μ and by a viscosity υ. A Maxwell solid behaves like an elastic solid over short time scales ($\upsilon \to \infty$), but flows like a fluid over long periods of time ($\mu \to 0$). This behavior is also known as elasticoviscosity.
2. When the tidal frequency is zero ($\sigma = 0$), the surface pressure variations become zero and equation (32) is no longer valid [*Siebert*, 1961]. However, it can be shown [*Correia et al.*, 2003] that this approximation can be used as long as $|\sigma| > n/100$.

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