

# Astrometry of Asteroids with Gaia.

## Simultaneous Determination of the PPN $\beta$ and Solar $J_2$

D. HESTROFER<sup>1</sup>, F. MIGNARD<sup>2</sup>, J. BERTHIER<sup>1</sup>, S. MOURET<sup>1</sup>  
 Email: hestro@imcce.fr  
<sup>1</sup>IMCCE, UMR CNRS 8028, observatoire de Paris, 75014 PARIS, France  
<sup>2</sup>Cassiopee, UMR CNRS 6202, observatoire de la Côte d'Azur, 06300 NICE, France

### 1. Introduction

The ESA cornerstone space-mission Gaia<sup>[1]</sup>, to be launched in 2011, will observe during its five years lifetime a huge number of celestial bodies up to magnitude  $V \leq 20$ . In addition to the '3D census of the Galaxy' this satellite will provide — from the survey character of the mission — multi-colour photometry and astrometric measurements for a large number of solar system objects (~500,000 mainly asteroids). The very high astrometric accuracy that will be achieved (on the sub-mas level), though one-dimensional, will provide dramatically improved asteroid orbits. This will enable the determination of the mass of some asteroids through analysis of mutual perturbation during close encounter, or provide information on their size and light scattering through analysis of the photocenter offset eventually combined with photometric measures.

Such high accuracy measures will also enable to derive global parameters of the perturbation of the orbit such as the Solar  $J_2$  and the relativistic effect on the perihelion advance. By providing global astrometry in an homogeneous reference sphere, Gaia observations of asteroids will also enable to indirectly link reference frames. A detailed simulation of the astrometry of subsets of asteroids (NEAs, MBAs, Trojans) has been performed yielding to realistic estimates of the observations distribution (in space and in time) and astrometric accuracy. We derive the formal precision that can be achieved for such global parameters.

Interestingly the PPN  $\beta$  and the solar quadrupole will be obtained directly and simultaneously, without any assumption on the internal structure of the Sun, and with a precision competitive with present or future experiments.

### 2. Observations of Solar System Objects

Due to the particular scanning law of the satellite, solar system objects are crossing the two telescopes' FOV in a (large) zone around their quadratures (between elongations L1 and L2, Fig.1). Estimation of the achievable astrometric precision is based on the focal plane simulation of the image from GIBIS+Pyxis simulators<sup>[4,5]</sup>. There is no particular pointing nor CCD reading

reading planned so far for these resolved and moving targets. Taking into account their size and motion degrades the astrometry, but it essentially remains a function of the target brightness (see Fig.2). These estimates are taken to be the precision for one 'observation' (i.e. crossing) in the along-scan direction (the astrometry in the perpendicular direction is much less precise). We consider a subset of asteroid-observable by Gaia including NE MBA, and Trojans (see Fig.3). contrast to the two others know and generally bright objects, the NEA population is based on a more extended simulated population following unbiased distributions<sup>[6]</sup>.

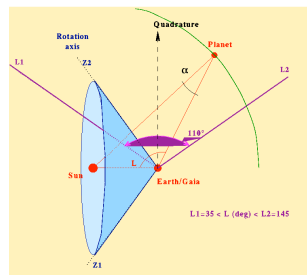


Fig.1: Scanning law of the satellite.

Fig.3: Perihelion precession (relativistic).

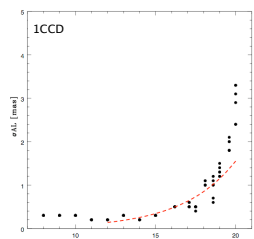
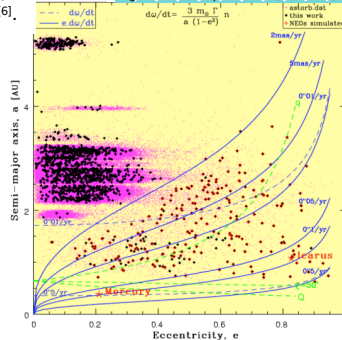


Fig.2: Estimated precision (AL) ranging from  $\sigma \approx 0.2$  mas for  $V < 12$  to  $\sim 5$  mas for  $V = 20$ .

### 3. Orbit Improvement

Partial derivatives wrt correction to the orbital elements ( $dq_i$ ) are obtained analytically following B&C<sup>[6]</sup> for the osculating elements, or numerically through variational equations for the position-velocity vector. We next compute the precision of the orbit correction  $\sigma(dq_i)$  for different asteroids. As given in Table 1, the orientation of the orbits should be given to  $\sim 10^{-7}$ – $10^{-8}$  degrees, the semi-major axis to  $\sim 5 \cdot 10^{-10}$ .

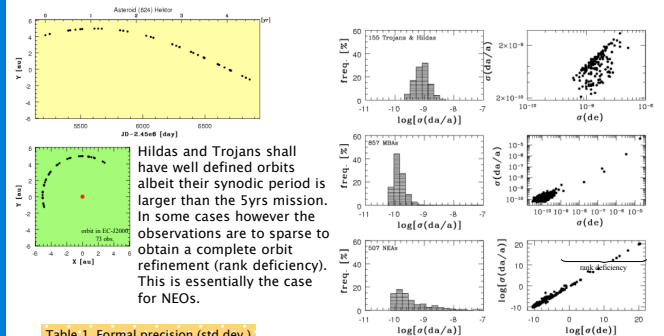


Table 1. Formal precision (std dev.) on orbit improvement.

	$dlo+dr$ [rad]	$dp$ [rad]	$dq$ [rad]	$e.dr$ [rad]	$de$	$da/a$	$d\Gamma+dJ_2$
NEO (Icarus)	1.0E-08	6.8E-09	9.3E-09	1.4E-09	3.3E-09	6.7E-10	3.2E-03
MBA (aver.)	2.3E-10	3.1E-10	3.1E-10	1.7E-10	1.4E-10	1.6E-10	2.1E-03
Trojan (aver.)	1.9E-09	4.2E-09	3.6E-09	8.8E-10	1.5E-09	1.1E-09	1.8E-02

### 4. PPN $\beta$ , solar $J_2$ & Reference Frame Link

In addition to parameters for individual asteroids (orbital elements, light scattering, mass, etc.), one can derive global parameters common to all targets that affect the observed positions. We consider the secular precession of the orbit from the combined effect of the relativistic perihelion drift (through  $\Gamma = (2+2\gamma - \beta)/2$ ) and the perturbation due to the Sun quadrupole moment  $J_2$ . Also a global rotation  $W$  and rotation rate  $dW/dt$  of the ecliptical dynamical reference frame w.r.t the ICRF can be introduced as additional parameter. Considering our set of asteroids we obtain the precision estimation in the Table 2 below.

The correlation between  $\Gamma$  and  $J_2$  is 0.76, the others drop to a few percent at most. One will hence be able to derive *directly and separately* the solar dynamical flattening  $J_2$  and the PPN  $\beta$  without any additional assumption on the internal structure of the Sun from helioseismology or measures of its external shape. These results appear to be competitive with present and future ground- or space-based experiments in the Solar system yielding  $\beta$  to  $10^{-3}$ – $10^{-4}$  and  $J_2$  to  $10^{-7}$ – $10^{-8}$  [e.g. 2,3].

Table 2. Formal precision on global parameters from 5 years mission and 1500 asteroids

	$\sigma(\beta)$	$\sigma(J_2)$	$\sigma(W)$	$\sigma(dW/dt)$
~1500 asteroids	6.3E-05	1.1E-08	x y z 2.5 2.5 11.0	x y z 2.7 1.7 5.3

### 5. Perspective

The precision of a single measure is conservative since an LSF fitting could be applied instead of the barycentre estimator, and one will have more than one single CCD observation during a transit (typically 10 for MBAs but less for fast moving NEAs). On the other hand the oblique illumination by the Sun will introduce a photocentre offset that can be important for the NEAs and not always fully modelled, introducing hence an additional noise or bias. Analysis of the photocentre offset and unmodelled perturbations by other massive asteroids shall be done in future work. Keeping in mind that only a subset of targets has been considered, a larger hypothetical set of 500,000 asteroids observed by Gaia will increase the precision on the global parameters. The solar quadrupole moment  $J_2$  could hence — by simple  $1/\sqrt{N}$  extrapolation — be determined *formally* to  $\approx 1.10^{-9}$ , i.e. with unprecedented precision. Similarly, an hypothetical rotation rate of the ICRF wrt the Dynamical RF would be determined on the  $\approx 1.10^{-12}$  rad/yr level.

### References

[1] M.A.C. Perryman, K. de Boer, C. Gilmore et al. (2001). *A&A* **369**, 339.  
 [2] J.G. Williams, S.G. Turyshev, and D.H. Boggs (2004). *Phys. Rev. Lett.* **93**, 261101.  
 [3] C. Will (2005). In: 100 Years of Relativity: Spacetime Structure — Einstein and Beyond.

[4] Babusiaux (2004). The Gaia Instrument and Basic Image Simulator. ESA SP-576, p.417.  
 [5] F. Arenou, C. Babusiaux, F. Chéreau, S. Mignot (2004). The Gaia On-Board Scientific Data Handling. ESA SP-576, p.335.  
 [6] Bottke, Morbidelli, Jedicke et al., (2002), *Icarus* **156**, 399.  
 [7] D. Brouwer & G.M. Clemence (1961). *Methods of Celestial Mechanics*.