Orbit Determination of Binary Asteroids.

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1. Introduction

In addition to the detection of an asteroid moon, the knowledge of the satellite's true orbit is of high importance to derive fundamental physical parameters of the binary system such as its mass. A new methodology for orbit determination of binary asteroids — and visual binaries in general — is proposed. It is based on Thiele-Innes method combined with a Monte Carlo technique. This method provides the full set of solutions (bundle or family of orbits, with the 7 orbital elements) even for a reduced number of observations.
The methodology also provides the detailed a-posteriori error distribution for such a non-linear inverse problem. Probability densities are obtained for the orbital elements and for the physical parameters, as well as for position predictions. It has successfully been applied to the orbit determination of known visual (stellar) binaries, and asteroids binaries. The method is presented here putting some emphasis on the size of the indeterminacy (in orbital elements, physical parameters, and ephemerides). We give some illustrations for some practical cases with different number of observations, and in particular for the KBO 2001-QT297, and the possibility to derive orbits from GAIA observations.

2. Thiele-Innes method & Monte-Carlo chain

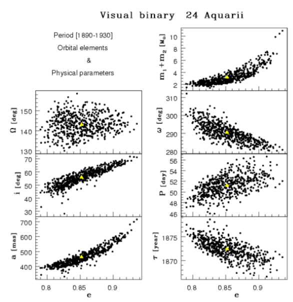
The method of Thile-Innes is straightforward and adapted to the most intricate cases of orbits determination [1]. It is also, as shown here, well adapted to orbit determination of binary asteroids. Being based on the fundamental equation relating eccentric anomaly E and areal constant C :

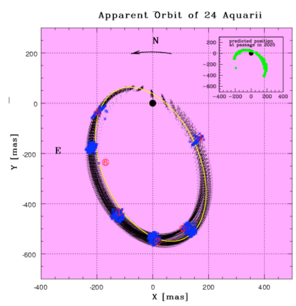
$$n\,(t_q-t_p) - \Delta_{pq}/C = E_q-E_p - \sin(E_q-E_p)$$
$$\Delta_{pq} = x_p y_q - x_q y_p = \rho_p \rho_q{}^* \cos(\theta_q-\theta_p) \qquad (1)$$

it directly provides a dynamical solution. Given three observations (ρ_i,θ_i) $i=[1,..3]$ and an orbital period P, Thiele's technique provides, if it exists, the unique Keplerian solution that verifies the 3 observations. Introducing next additional observations and observational noise one will derive all possible solutions that fit the observations. Thus the only free parameter being explored is the orbital period that -- in the case of binary asteroids with periods of a few days -- can often been infered after one observing run. Such procedure, which is similar to the 'statistical ranging' technique of [2], hence provides the true a-posteriori density distribution of the foreseen orbital and physical parameters. The method is tested on the well-known apparent orbit of the visual binary 24 Aquarii (=ADS 15176), for which we have extracted some observational data in the table below.

Observations of 24 Aquarii

Date [yr]	1890.75	1900.74	1910.40	1921.66	1926.64	1930.48
ρ [arcsec]	0.45	0.55	0.51	0.22	0.20	0.29
θ [deg]	164.5	181.9	192.2	231.1	100.7	144.3

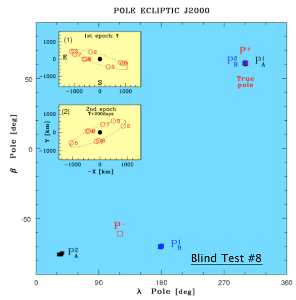


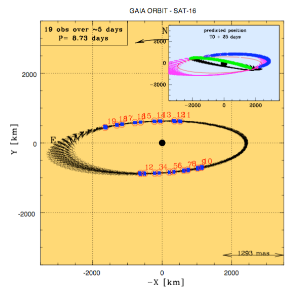


The orbits thus derived shows the distribution of the parameters and the bundle of orbits for a rms error of ~30 mas. Knowledge of the system's parallax yields the total mass $m=3M_o$ (these main-sequence stars are expected to be a little more massive than the Sun). By showing the whole range of possible values, the method is especially useful in cases where the period is so long that only a fragment of the orbit has been recorded. In addition the uncertainty region for the predicted periastron passage in 2025 is given.

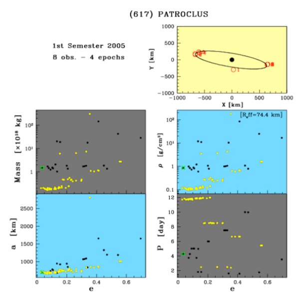
3. Binary asteroids

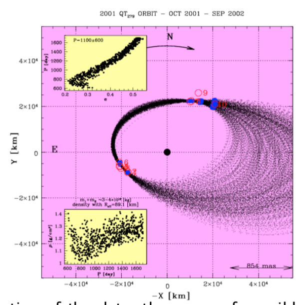
The method can be applied to binary asteroids [3]. Having typical orbital periods of a few days one can gather a few observations during an observing run. One can also infer the position of the satellite and its uncertainty during a future stellar occultation event. Due to their faintness, some systems are difficult to observe (trojans, Kuiper-belt) yielding sparse data. Preliminary orbit computation for systems observed with GAIA can also be obtained.





Blind Test #8. In contrast to stars, having at least two run at different epochs enables one to unambiguously derive the orbital pole.
GAIA simulation. The satellite GAIA, by observing asteroids brighter than V<20, shall observe or discover binary system. The distribution in time of the observations is determined by the particular scanning law of the satellite.





617 Patroclus. Given the temporal distribution of the data, the range of possible orbits is large, either clockwize or conter-clockwize, comprising different periods. Only supplementary data yield the right range of period and mass as will be shown in [4].
2001 QT_{297}. Observations from [5]. The range of possible orbits is still large. Given the absolute magnitude H=5.5, densities of the order of 1 g.cm^{-3} are only possible for high albedo of ~0.25.

4. Secular effects

Knowledge of the orbital elements provides, through Kepler's third law, the total mass of system. However asteroid being generally elongated bodies, the most important terms in the gravity field are the C20(=$-J_2$) and C22 (which latter has has little secular effect with typical orbital/spin period ratio). Neglecting the dynamical flattening J_2 introduces a bias in the mass determination that is generally negligible

$$n^2a^3\,(1-3(Re/a)^2\,J_2) = G\,(m1+m2) \qquad (2)$$

In the particular case where the primary possess at least two satellites, of different orbital periods, one can simultaneously derive the mass and J_2.

A (a~1400km, P~3.6d) & B (a~730km, P~1.4d)

J_2	0.00	0.20	**0.22**	0.25	0.30
m_A	15.253	15.086	**15.069**	15.044	15.003
m_B	15.754	15.131	**15.070**	14.978	14.825

With a single satellite, the effect of the J2 (~0.1-0.2) shall be observed on the secular precession of the orbital plane for non equatorial orbits, or on the periastron drift for eccentric orbits. Typical periods for a complete precession are of ≈ 3-5 years for the MBBs (~ 10^3 revolutions) and much more for the known KBBs.

References:
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[4] F. Marchis et al. (2005). DPS 37, #14.07.
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