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Astrometric reduction of CCD observations of planetary satellites without reference star: application to Saturn' satellites

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Abstract

We present an astrometric reduction for the case of inter-satellites measurements when no astrometric stars is present on the frame. We discuss precisely all significant astrometric corrections. Some of these corrections are often neglected, but we show they can not be negligible. The reduction presented here allows to give coordinates apart from a scale factor and from a rotation. These coordinates are astrometric because no astrometric consideration is necessary to use them, even if one wants to touch up the calibration. The good accuracy obtained is needed for future use of these data to improve the dynamical models.

Résumé

Nous présentons une réduction astrométrique pour le cas de mesures inter-satellites sans étoile de référence sur l'image. Nous discutons précisément toutes les corrections astrométriques significatives. Quelques une de ces corrections sont souvent négligées, mais nous montrons qu'elles ne peuvent l'être. La réduction présentée ici permet d'obtenir des coordonnées à une rotation et à un facteur d'échelle près. Ces coordonnées sont astrométriques car aucune considération astrométrique n'est nécessaire pour les utiliser, même si on souhaite modifier la calibration. La bonne précision obtenue est nécessaire pour les utilisations futures de ces données dans l'amélioration des modèles dynamiques.

1 Introduction

Since 1990, and apart from the observations of the mutual phenomena in 1995 (Arlot & Thuillot [1993], Thuillot et al [2001]), the observations of Saturn satellites are CCD ones. The situation is nearly the same for other planetary satellites. Often, CCD observations of planetary satellites are published in pixels and no astrometry are really done upon them. The problem is that the scale of these frames are rather small. So, most often, there is no reference stars which

Table 1: Differential effects

effect	correction
refraction	$1''$ ($z \leq 70°$)
stellar aberration	$0.''04$
central projection	$0.''03$
light-travel time between satellites	$0.''025$
topocentric parallax	$0.''002$

would allow to do an astrometric reduction. In order to estimate the quality of these observations, the observers use the positions of the satellites itself to do a reduction. They give an indicative value of the scale factor and of the angle of orientation. These values are difficult to interpret. Furthermore, we have found that they are affected by some errors or neglected effects.

Here we intend to extract the astrometric data from these observations. So, we described all significant astrometric corrections to be done. The corrections presented here are standard but often neglected elsewhere. And then, we give a rather detailed discussion of the different steps of the reduction.

In order to illustrate the reduction, we give the description applied to the observations done in 1995 at the Laboratório Nacional de Astrofísica at Itajubá in Brazil . The reader can find more details about these observations in (Vienne et al [2001]). More generally, most of our comments concerns the Saturnian system, because we have particularly studied this system, and because it is in this system we have found some misunderstandings. But, of course, the reduction itself can be applied upon other planetary systems.

2 Local deformations

The frames registered and measured in pixels are not directly comparable with ephemeris, because they have been deformed by some local effects. So, we have to take into account the differential astrometric corrections given in Table 1.

The values given in the right column are indicative; they correspond to a maximum value in the Saturnian system, for a differential effect applied to a standard frame $400''$ large (except for the central projection, see below). These astrometric corrections are classical and so, they are generally well described in any astrometry handbook, but we want to insist upon them because some confusions exist. These confusions can surely be explained by the fact that, in the past, the precision level was about $0.''20$ for satellite astrometry. For their data set of CCD frames, Harper et al [1997] have taken into account the "differential parallax, the aberration and refraction". So, the reasons why some corrections are done or not are not so clear.

For the refraction, the effect, even differential, is so large that no astronomer can ignore this correction. Note that the formulae used corresponds to normal conditions. That is sufficient for the differential effect. But for absolute coordinates, it is necessary to take into account the pressure, the temperature and the wavelength of the light.

For the stellar aberration, Harper et al [1997] do this correction but some other astronomers said that this effect affects equally the positions of stars, Saturn and the satellites. Then, in this case, they ignore a correction up to

$0.''04$.

The correction of the central projection corresponds to a deformation of the coordinates lines $(\Delta\alpha \cos\delta, \Delta\delta)$. The drawing of these lines on a frame are not rectangular. Although, that does not correspond to a deformation of the frame itself, we design it with the generic name 'local deformations'. When a frame is measured (in pixels for example), the coordinates are linearly linked to the tangential coordinates X and Y. They are defined in the tangential plan of the celestial sphere at a point C which is generally the center of the frame. Let (α_C, δ_C) the position of C on the celestial sphere, and let a satellite referred to C on a celestial map by the differential coordinates $(\Delta\alpha \cos\delta_C, \Delta\delta)$, we can compute X and Y, with:

$$\begin{aligned} X &= \Delta\alpha \cos\delta_C - \Delta\alpha\Delta\delta \sin\delta_C + \cdots \\ Y &= \Delta\delta + \frac{1}{2}(\Delta\alpha)^2 \sin\delta_C \cos\delta_C + \cdots \end{aligned}$$

The correction $(X - \Delta\alpha\cos\delta, Y - \Delta\delta)$ is about $s^2 \tan\delta_C$ (s is the separation angle), whereas the corrections for refraction and aberration are proportional to s. Consequently, the maximum value ($0.''3$ for $s = 400''$ et δ_C =23°) is rarely reached and is not a typical value. For example, if we suppose that C is at the center of the frame of the observations of Harper et al [1997], the correction reaches the maximum value $0.''022$. For the brazilian observations presented in Sect.4 as an example , the effects reach only $0.''004$ because Saturn is near the equator (δ_C=−4°). Note that, to apply such a correction, we have to know the position of the focal point on the frame. Supposing arbitrarily, as it is done sometimes, this point to a satellite taken as reference, is often incorrect and useless. As C does not correspond to a physical object, we do not know (α_C, δ_C). We estimate (α_C, δ_C) by an iterative procedure beginning by applying the reduction described in Sect.3 without the present correction. Two iterations are enough to obtain a good estimation.

Until now, the light-travel time between the satellites of Saturn seems to us to be ignored. In this case, the assumption is equivalent to consider that all satellites are at the same distance from the observer (the distance between the observer and Saturn). The maximum error on the time argument T for a given satellite is the light-travel time, τ, from Saturn to the satellite. We have $\tau \leq a/c$ where a is the semi major axis and c the velocity of the light. So, τ is maximum ($\sim 12s$) for Iapetus, the outermost satellite in the present work. If we take $n \times a$ for the velocity of the satellite, where n is the mean motion of the satellite, the difference Δp in position is then proportional to $\sqrt{a}$. Δp is still maximum for Iapetus and the value is about 39 km that corresponds to $0.''006$ as seen from Earth. Giving a similar argument, Harper & Taylor [1994] conclude that this effect is entirely negligible. In their following works, this correction is then neglected. But, here, we point out that the expression of Δp proportional to $\sqrt{a}$ is not complete because we have also to consider the velocity of Saturn itself. More precisely, we have to consider this velocity in an inertial frame because the velocity of the observer is taken into account in the stellar aberration. Taking 10 km/s for the velocity of Saturn, all the Saturnian system has moved of about 120 km during the time $\tau = 12s$. Finally, we find that this effect has two parts: one proportional to $\sqrt{a}$ (roughly, the

Table 2: Maximum effects of neglecting the light-travel time τ between the satellites of a planetary system.

satellite	τ	Δp (direct)		Δp (non direct)	
	s	km	mas	km	mas
Phobos	0.03	0	0	1	2
Deimos	0.08	0	0	2	5
Io	1.4	24	8	18	6
Europa	2.2	31	10	29	10
Ganymede	3.6	39	13	47	15
Callisto	6.3	51	17	83	27
Mimas	0.6	9	1	6	1
Enceladus	0.8	10	2	8	1
Tethys	1.0	11	2	10	2
Dione	1.3	13	2	12	2
Rhea	1.8	15	2	17	3
Titan	4.1	23	4	40	6
Hyperion	4.9	25	4	48	8
Iapetus	**11.9**	**39**	**6**	**115**	**19**
Phoebe	43.0	74	12	417	67
Miranda	0.4	3	0	3	0
Ariel	0.6	4	0	4	0
Umbriel	0.9	4	0	6	0
Titania	1.4	5	0	10	1
Oberon	1.9	6	0	13	1
Triton	1.2	5	0	6	0
Nereid	18.4	20	1	101	5

direct effect), and the second proportional to a (the non direct effect due to the planet). This separation means that we have to compute n positions of Saturn for a single frame containing n satellites, each one corresponding to a different date $T - \tau_i$ itself computed by an iteration process. In practical computation, that is naturally done by using geocentric positions of a satellite and then, the separation of Δp in two parts is not explicit. The separation was done here only to show the approximation of Harper & Taylor. We gather in Table 2, the maximum value of the light-travel time correction for the major satellites of the solar system. For Iapetus, we find a direct effect of $0.''0063$ as Harper & Taylor [1994], nevertheless, the second part, which was neglected in the past, is three times greater.

The topocentric parallax is taken into account by most of the observers, but, for inter-satellite measurements and for Saturn, the effect is very low ($\leq$ 4mas).

For absolute coordinates, this last effect reaches $1''$, that is 6380 km seen at 8AU. So, any error on the position of Saturn (or on the one of the observer) smaller than $1''$ would have an influence upon the coordinates smaller than the one from topocentric parallax. The positions of Saturn is given by the ephemerides SLP96 from the "Institut de mécanique céleste (IMCCE)" (available at ftp://ftp.bdl.fr/pub/ephem/sun/slp96/) found on the VSOP87 planetary theory (Bretagnon & Francou [1988]). The precision on the positions of Saturn is about $0.''4$ which is enough here. In fact, our ephemerides give, theoretically, the position of the gravity center of the Saturnian system. Supposing this point as the center of Saturn leads to a difference of about 290 km (mainly due to Titan). This error is 20 times smaller than the topocentric parallax, and so, it is also acceptable for our inter-satellites reduction.

At last, let us note also two corrections we have not taken into account. The first one is due to the gravitational light deflection from the Sun. For a geocentric elongation of Saturn from the Sun equal to 20° (and still for $s = 400''$), the differential effect is $0.''00014$ (0.14 mas). For the observations of Sect. 4, this elongation is greater than 85°, and the correction is lesser than $0.''009$ mas. The second correction is due to the gravitational light deflection from the planet itself. The gravitational field is lesser than the one of the Sun, but the observed objects are nearer. Furthermore, there is not really a differential effect since the difference for two satellites is generally of the same order of magnitude than the effect itself. We find for Iapetus that this effect is lesser than 0.02 mas.

3 The procedure of the reduction

The procedure of the reduction is visualized on Figure 1. In order to have a simple drawing, it appears only one frame. But, the procedure is in fact applied only once for all frames of a given series. Generally, a series of frames corresponds to one campaign of observations and covers several nights. From the diagram described below, we then suppose that the receptor has been mounted in the same way for all the frames of the series. The procedure is the following one:

- Ephemerides: we use ephemerides for the saturnicentric positions of the satellite:TASS1.7 (Vienne & Duriez [1995], Duriez & Vienne [1997]). The position of Saturn itself is given by the ephemerides SLP96 (see above). The time argument is the Terrestrial Time (TT).

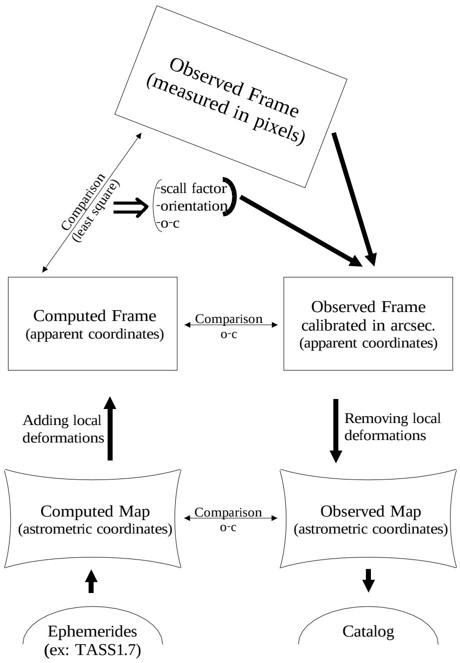

Figure 1: Diagram of the reduction

- Computed map: from the ephemerides, and at a given date corrected for the light-travel time between the satellites, we are able to draw up a map of the objects. This map is under the form of astrometric coordinates on the celestial sphere (α, δ in the J2000 equator and equinox system).

- Computed frame: we apply to these coordinates the corrections described in Sec. 2 (except the light-travel corrections already done). These local deformations are taken into account in order to obtain a frame as a receptor could have registered. This is this frame which can be directly compared to the frame really registered and measured. Its form is a table of positions X_c and Y_c, given in pixels.

- Observed frame: each frame is measured, so, in this procedure, we use it as a table of positions (x_p, y_p), given in pixels.

- The least square procedure: the coordinates x_p and y_p of the observed frame are linearly linked to the tangential coordinates X and Y of the computed frame,

$$\begin{pmatrix} a & b \\ -b & a \end{pmatrix} \begin{pmatrix} x_p \\ y_p \end{pmatrix} - \begin{pmatrix} X \\ Y \end{pmatrix} = 0$$

  If we put $X = X_c$ and $Y = Y_c$ in the left-hand member of this relation, we can compute an estimation $\widetilde{a}, \widetilde{b}$ of a and b by a least square procedure. We deduce the scale factor $\rho = \sqrt{\widetilde{a}^2 + \widetilde{b}^2}$ and the orientation of the receptor (the angle from the x_p-axis of the camera to the true equator)

$\varphi = \arctan(\widetilde{b}, \widetilde{a})$. Note that, because of the linearity of the relation between (X, Y) and (x_p, y_p), no iterative procedure is necessary. It is not true for the direct determination of ρ and φ.

- Observed frame (calibrated in arcsec.): we only apply the transformation:

$$\begin{pmatrix} X_o \\ Y_o \end{pmatrix} = \begin{pmatrix} \widetilde{a} & \widetilde{b} \\ -\widetilde{b} & \widetilde{a} \end{pmatrix} \begin{pmatrix} x_p \\ y_p \end{pmatrix}$$

  This transformation simply corresponds to a calibration of the frame. The coordinates are of the same nature than the previous ones. In other words, these coordinates would be "probably" (by means of the least square procedure) the ones directly measured in pixels by the same camera but mounted in such way that the x_p-axis corresponds to the true equator of the date, and that one pixel corresponds to one second of degree on the celestial sphere.

- Observed map: we apply once more time, but inversely, the local deformations, in order to get the coordinates (α, δ) of each measured object.

- Catalog: the coordinates (α, δ) are astrometric, but the absolute part (α_C and δ_C) does not come from the observations. So, to avoid any confusion, we publish the observations in inter-satellites form. That is $(\Delta\alpha \cos\delta, \Delta\delta)$, or more precisely, $(\alpha_o - \alpha_r)\cos\delta_r$ and $\delta_o - \delta_r$. The index $_o$ is for the object satellite, and the index $_r$ for the reference satellite.

This procedure is efficient for the eight major satellites of Saturn, or more generally for all objects for which we have ephemerides. But, in the least square procedure, only the positions of Tethys, Dione, Rhea and Titan are used to calibrate the frame because these satellites have the best ephemerides, probably affected by the smallest systematic effects.

We have tried to determine the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ instead of $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$. We have found that $a \simeq d$ and $b + c \simeq 0$, the difference being inside the probable error. Furthermore, the geometrical interpretation of such transformation would be more difficult to do.

It is interesting to note that, in the diagram of the Figure 1, each line of "o-c" corresponds to only one distribution of the residuals, the one issued from the least square procedure. It is obvious for the o-c between the 'Computed Frame' and the 'Observed Frame (in arcsec.)" because both observed frames are linearly linked. Between the 'Computed map' and the 'Observed map', we have also the same distribution of o-c because despite their non linearity, the local deformations are small. So, adding and removing theses deformations are almost equivalent (at the second order of these corrections). Consequently, if we do a least square procedure directly between both maps, we find the same o-c, $d\varphi \equiv 0$ and $d\rho \equiv 0$. Then, it means also that if we compute the map in any other way, for example with other ephemerides, we can touch up φ and ρ without doing again all the procedure.

For any catalog, it is important to know the method of reduction to allow future uses of the observations. For example, the scale factor depends upon the mass of Saturn used. So, it is evident that these observations cannot be

Table 3: Statistics of the o-c (in mas) from the comparison with the ephemerides TASS. N give the number of observations used. These residuals are relative to S5 (if S5 is absent of the frame, we use S6, and so on with the ordering list: S5, S6, S4, S3, S2, S1, S8 and S7). The last lines give the global residuals of S3, S4, S5 and S6 because the computed positions of these satellites have been used in the reduction.

		$\Delta\alpha \cos\delta$		$\Delta\delta$	
satellites	N	means	r.m.s.	means	r.m.s.
S1-Mimas	216	-23	85	-1	78
S2-Enceladus	865	+11	92	-6	68
S3-Tethys	2151	+3	79	+3	65
S4-Dione	1466	-8	64	-1	57
S6-Titan	460	+19	81	+11	67
S7-Hyperion	324	-93	152	-39	128
S8-Iapetus	524	-108	141	+10	69
S3 S4 S5 S6	4077	+1	74	+2	62

used to determine the mass of Saturn. On the contrary, they could be useful to determine the eccentricity of Tethys via the positions of Mimas and Tethys which are rather numerous. Champenois & Vienne [1999a], [1999b] have shown that the knowledge of this eccentricity is important to understand the evolution of the resonance of the Mimas-Tethys system. Then, the method of reduction must appear explicitly with the catalog. The user will judge itself and under his own responsibility which informations he can extract from these data. It is why, we add at the end of the catalog, some lines which indicate how the reduction have been done and which corrections have been applied to get the coordinates. Specially, a column "catalog" indicates the source of the theoretical positions used to reduce the plates. Here, we indicate S3, S4, S5 and S6 from TASS1.7. But if we have been able to use Hipparcos stars for our reduction, we would indicate "Hipparcos" !

Two other columns give the scale and the orientation of the frame. Note that the orientation refers to the true equator of the date because the calibration parameters are issued from a direct comparison between the observed frame and a "computed frame". But as we have seen above, it is possible to touch up φ and ρ, directly by comparing the "observed map" and the "computed map" (then given in the J2000 system). For example, a correction of the parameters φ and ρ could be useful to reduce dynamical theory. Another example, a new value for the mass of Saturn can be considered throughout a corrected value of ρ.

4 Application

We have applied our reduction upon the observations done in 1995 at the Laboratório Nacional de Astrofísica at Itajubá in Brazil . More details about the analysis can be found in (Vienne et al [2001]). The statistics is shown in the Table 3. The dispersion of the observations is about 0."07, and the bias of some milli-arcseconds. As Tethys, Dione, Rhea and Titan have been used in the fit, this fact is true for these satellites taken separately or globally. The positions of the satellites not used in the calibration have a good quality: 0.”08 for the 216 positions of Mimas, 0.”14 for Hyperion (324), 0.”11 for Iapetus (524). For comparison, Shen et al [2001] have analyzed CCD observations over the 1990-1997 period from Qiao et al [1999], Harper et al [1997], [1999]. They give the following residuals: 0.21 (Mimas), 0.”16 (Hyperion), 0.”16 (Iapetus), corresponding respectively to 57, 218 and 230 positions.

5 Conclusion

We have presented an astrometric reduction for the case of inter-satellites measurements (CCD receptors) without astrometric star. So, the astrometric corrections of the positions have differential effects only. Some of these corrections are often neglected, but a rather detailed discussion shows they can be not negligible. Apart from the refraction 1”. ($z \leq 70°$), the stellar aberration (0”.04) and the topocentric parallax (0”.002) which, usually, are taken into account, we consider also the central projection (0”.03) and the light-travel time between the satellites (0”.025). This last effect is found three times greater than the estimation based upon a description neglecting the velocity of Saturn.

The reduction presented in the present work allows to give coordinates apart from a scale factor and from a rotation. But all the astrometric corrections are done. So, these positions are really astrometric ones in that meaning that, no astrometric consideration is necessary to use them, even if one wants to touch up the calibration.

This reduction has been applied for the Brazilian observations of 1995 (Vienne el al [2001]), and also in more recent observations (Peng et al [2001]).

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